Strength of Tubular Samples and Tubular Cracked Junction Under Combined Loads

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The strength of tubular sample loaded by only one load or simultaneously with several loads is analyzed. Critical stresses and critical loading parameters were introduced and discussed in the case of nonlinear power law behavior. On the basis of principle of critical energy eqs. for superposition of loading effects have been obtained. The influence of deterioration due the cracks has been introduced. The obtained theoretical results were applied to tubular branch junctions and verified against data from literature.

Keywords: Critical stresses; simultaneous loadings; tubular specimen; tubular branch junction; principle of critical energy

Pressure equipment and pipes, as well as tubular nozzles, often are simultaneous loaded by internal pressure, axial force, bending moment and torsional moment. Tubular nozzles of pressure equipment may be: perpendicular, inclined, oblique or tangential to the shell surface. In the junction area between nozzle and shell the stresses often exceed the yield stress. At stresses below the yield stress the superposition of stresses due to different loads may be done by algebraic summation. If the stress exceed the yield stress one can not use the algebraic summation. In this case the principle of critical energy (PCE) must be used [1; 2]. In correlation with this principle one uses critical values of forces, bending moments, stresses etc.

In general, the critical stress, as well as the critical parameter (pressure, axial force, bending moment etc.) is that value of stress or of parameter that determines the taking out of use or the destruction of a sample or a mechanical structure. The value of critical stress or parameter can be chosen; for example the value which determines the yielding of material (corresponding to yield stress) or the value which determines the failure (corresponding to ultimate stress). The critical stress or critical parameter is marked by subscript *cr*. For example σ_{cr} is the critical stress, F_{cr} is the critical axial force etc.

Until recently, strength calculation of structures *static loaded* refers to materials with linear - elastic behavior. Components and structures are usually calculated and constructed according to stress principles: the equivalent stress must be less or equal to an allowable value. But many mechanical structures undergo defects or cracks, as well as welding residual stresses. The problem is how to calculate the mechanical structures taking into account: - the influence of cracks and residual stresses; - the nonlinear behaviour when stress is higher the yield stress.

A first way consists in resort to the principle of critical energy [1; 2] which introduces the concept of specific energy participation (the energy of a unit volume, J / m^3 or of a unit mass J/kg).

In the case of fatigue loading it was found experimentally that when specimens without cracks (fig. 1) are under load, fatigue strength begins to decrease after a number of cycles $N > N_f$, while at specimens with cracks, fatigue strength begins to decrease at $N > N^*_f$, where $N^*_f < N_f$.

One can see that when $N \leq N^*_f$, the critical crack length a_{cr} is constant, while when $N_f > N^*_f$, the crack will increase continuously with an increasing number of load cycles [3]. In the case of the materials under study (fig. 1), the fatigue strength begins to decrease when the number of cycles exceeds $N_f = 1000$ in specimens without cracks and down to $N^*_f = 100...300$ cycles in specimens with cracks. When $N < N_f$ and at $N < N^*_f$, rupture is essentially quasi-static.

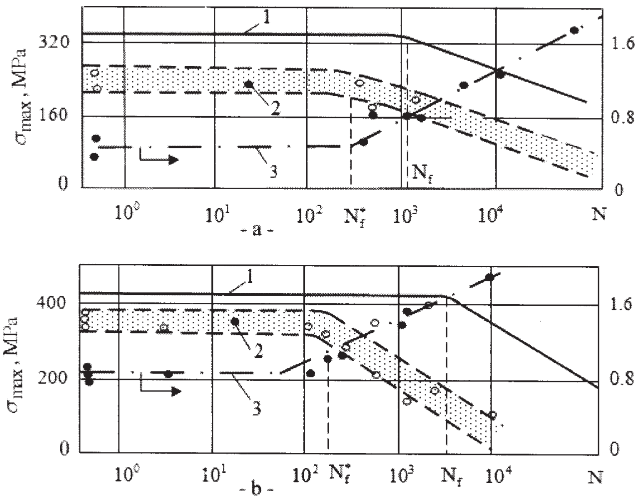

Fig. 1. Fatigue curve for test specimens without cracks (1) and with cracks (2) from aluminum allois [3]: Δ20-1 (a) and AlMg6 (b). Curve 3 refers to the crack half-critical length c_{cr}

Critical stresses and critical loads

We consider the nonlinear, power law, behavior of the material, under normal stress, σ, and shear stress, τ, given by the eqs.,

$$\left.\begin{aligned} \sigma &= M_\sigma \cdot \varepsilon^k ; \\ \tau &= M_\tau \cdot \gamma^{k_1} , \end{aligned}\right\} \qquad (1)$$

where ε is the strain; γ is the shear strain; M_σ, M_τ, k and k_1 are material constants.

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In recent works [4-6] on the basis of the principle of critical energy there have been proposed the following relations for critical stresses of tubular specimens with cracks:

$$\left.\begin{aligned}\sigma_{cr}(a;c) &= \sigma_{cr}\cdot\left[1-D(a_\sigma;c)\right]^{\frac{1}{\alpha+1}};\\ \tau_{cr}(a;c) &= \tau_{cr}\left[1-D(a_\tau;c)\right]^{\frac{1}{\alpha_1+1}},\end{aligned}\right\} \quad (2)$$

where the total deterioration $D(a_\sigma; c)$ depends on the crack depth $a \equiv a_\sigma$ and the crack length 2c in the direction perpendicular to the direction of the normal stress σ, while deterioration $D(a_\tau; c)$ depends on the depth of the crack $a \equiv a_\tau$ and the crack length 2c in the direction of shear stress τ.

The relationships proposed in the literature [7-9] for yield loading in tubular cylindrical specimens with cracks, generally can be written as in eq. (2), namely

$$Y_L = Y_y\cdot\left[1-\left(D(a;c)\right)\right], \quad (3)$$

where Y_L is the limit load of the cracked tubular specimen; Y_y is the limit load of the crackless tubular specimen and $\alpha = \alpha_1 = 0$.

Based on this relationship, yield loads in tubular specimens with cracks (F_L; $M_{b,L}$; p_L) were written in table 1 as a product of the yield load of the specimen without cracks (F_y; $M_{b,y}$; p_y) and a bracket comprising damage D(a; c). The last column lists the constant from damage expressions resulting from papers [7÷9].

To see the effect of crack location, for the external cracks, one uses the outer radius, R_2, instead of mean radius, R_m, when calculate the limit loadings, F_y, M_y and p_y. For thin walled cylinders the results are approximately the same; the effect of crack location is not significant for shorter cracks (smaller values of θ / π) [7].

On the basis of Morozov's criterion of rupture for cracked sample [10;11] at $\sigma \le \sigma_y$ one may write,

$$\sigma_{cr}(a;c) = \sigma_{cr}\cdot\left(1-c/c_{cr}\right)^{0.5}, \quad (4)$$

where $D(c) = c / c_{cr}$ with 2c the length of the crack and $2c_{cr}$ its critical value. In this case $\alpha = 1$.

Similarly, on the basis of Andreikiv criterion [12], the following failure criterion was written,

$$\varepsilon_{cr}(c) = \varepsilon_{cr}\cdot\left[1-\left(c/c_{cr}\right)^m\right]^{1/m}, \quad (5)$$

where $\varepsilon_{cr}(c)$ and ε_{cr} are the effective and the critical strain, while m is an exponent determined experimentally.

The empirical relations (4) and (5) are similar to theoretical relation (2).

Structure strength calculation based on allowable stresses

a. *Classical strength calculation methods* currently used in official norms are deterministic methods and are based on the condition,

$$\sigma_{eq} \le \sigma_{al}, \quad (6)$$

Table 1
EQS. FOR YIELD LOADS IN TUBULAR SPECIMENS WITH CRACKS [7÷9] WRITTEN AS IN EQ (3)

No.	Eq. of limit load of cracked tubular specimen, under a single load	Eq. for deterioration	Constants
1	$F_L = F_y\cdot\left[1-\left(D_1(a;c)\right)\right]$	$D_1(a;c) = -\left[A_1\cdot\left(\frac{a}{t}\right)+A_2\cdot\left(\frac{a}{t}\right)^2\right]$	$A_1 = 0.0662-0.038\left(\frac{\theta}{\pi}\right)-0.96\left(\frac{\theta}{\pi}\right)^2$ $A_2 = -0.0598-1.525\left(\frac{\theta}{\pi}\right)+1.4267\left(\frac{\theta}{\pi}\right)^2$ $F_y = 2\cdot\pi\cdot R_m\cdot t\cdot\sigma_y$
2	$M_{b,L} =$ $= M_{b,y}\cdot\left[1-\left(D_2(a;c)\right)\right]$	$D_2(a;c) = -\left[B_1\cdot\left(\frac{a}{t}\right)+B_2\cdot\left(\frac{a}{t}\right)^2\right]$	$B_1 = 0.0741-0.1693\left(\frac{\theta}{\pi}\right)$ $B_2 = -0.0863-1.0127\left(\frac{\theta}{\pi}\right)$ $M_{b,y} = \left(4R_m^2\cdot t\right)\cdot\sigma_y.$
3	$p_L = p_y\cdot\left[1-\left(D_3(a;c)\right)\right]$	$D_3(a;c) = -\left[A_1\cdot\left(\frac{a}{t}\right)+A_2\cdot\left(\frac{a}{t}\right)^2\right]$	$p_y = \frac{2\cdot t}{R_m}\cdot\sigma_y$
4	$p_L = p_y\cdot\left[1-\left(D_4(a;c)\right)\right]$	$D_4(a;c) = \left[\frac{\theta}{\pi}\cdot\frac{a}{t}+2\sin^{-1}\cdot\left(\frac{a}{t}\cdot\frac{\sin\theta}{2}\right)\right]$	$p_y = \frac{2}{\sqrt{3}}\cdot\frac{t}{R_m}\cdot\sigma_y$
5	$p_L = p_y\cdot\left[1-\left(D_5(a;c)\right)\right]$	$D_5(a;c) = -\left[C_1\cdot\left(\frac{a}{t}\right)+C_2\cdot\left(\frac{a}{t}\right)^2\right]$	$C_1 = 0.0462-0.0589\cdot\rho-0.013\cdot\rho^2$ $C_2 = 0.0395-0.3413\cdot\rho+0.0652\cdot\rho^2$ $p_y = \frac{t}{R_m}\cdot\sigma_y$ $\rho = c/\sqrt{R_m\cdot t}$
6	$p_L = p_y\cdot\left[1-\left(D_6(a;c)\right)\right]$	$D_6(a;c) =$ $= \left[\frac{a}{t}\cdot\left(1-\frac{1}{\sqrt{1+0.34\cdot\rho\cdot\frac{a}{t}+1.34\cdot\rho^2\frac{a}{t}}}\right)\right]$	$p_y = \frac{2}{\sqrt{3}}\cdot\frac{t}{R_m}\cdot\sigma_y$
7	$p_L = p_y\cdot\left[1-\left(D_7(a;c)\right)\right]$	$D_7(a;c) = -\left[D_1\cdot\left(\frac{a}{t}\right)+D_2\cdot\left(\frac{a}{t}\right)^2\right]$	$D_1 = -0.1429+0.134\cdot\rho-0.043\cdot\rho^2$ $D_2 = 0.1587-0.5928\cdot\rho+0.1131\cdot\rho^2$ $p_y = \frac{t}{R_m}\cdot\sigma_y$ $\rho = c/\sqrt{R_m\cdot t}$

where σ_{eq} is the equivalent stress, calculated by using a strength theory (Tresca or von Mises theory). The allowable stress in this case $\sigma_{al} = \sigma_{cr} / c_\sigma$, where $c_\sigma > 1$ is the safety coefficient.

On principle, this method of strength calculation can be applied to structures with cracks, too. The strength requirement for a structure with cracks may be written as,

$$\sigma_{eq} \leq \sigma_{al}(a;c), \qquad (7)$$

where σ_{al} (a;c) is the allowable stress of the cracked specimen,

$$\sigma_{al}(a;c) = \sigma_{cr}(a;c)/c_\sigma ,$$

where σ_{cr}(a;c) results from the first eq. (2).

b. *The calculation method based on the principle of critical energy* shows that the loading state must meet the condition [6],

$$P_T^* \leq P_{al}, \qquad (8)$$

where the total participation versus the allowable state has the expression,

$$P_T^* = \sum_i \left(\frac{\sigma}{\sigma_{al}}\right)_i^{\alpha+1} \cdot \delta_{\sigma_i} + \sum_j \left(\frac{\tau}{\tau_{al}}\right)_j^{\alpha_1+1} \cdot \delta_{\tau_j}, \qquad (9)$$

where $\sigma_{al} = \sigma_{cr} / c_\sigma$ and $\tau_{al} = \tau_{cr} / c_\tau$ are the allowable stresses of uncracked specimen, while $c_\sigma > 1$; $c_\tau > 1$ are the safety coefficients with respect to the normal and shear stress, respectively.

The allowable participation, P_{al}, is given by the eq. [6],

$$P_{al} = 1 - D_T^* - P_{res}^*, \qquad (10)$$

where $D^*_T(t)$ is the total damage with respect to the allowable state and P^*_{res} is the specific energy participation corresponding to the residual stress, with respect to the allowable state.

Strength of tubular sample

In practice, piping systems as well as pressure equipment with nozzles, are always subjected to combined pressure and system loadings (bending moment, torsional moment, forces…), thus the studies need to be carried out of combined loading; it needs often to consider the superposition of the deteriorations due to cracks.

Un-cracked tubular structures. General case

Consider a certain structure whose material behaves according to relations (1). Under a group of loads such as F_i(i=1; 2; 3 ... n), *the total participation of specific energies* introduced into the structure material is written as [1],

$$P_T = \sum_i \left(\frac{F}{F_{cr}}\right)_i^{\alpha+1} \cdot \delta_F, \qquad (11)$$

where $F_{cr,i}$ is the critical value of the generalized load F_i, while $\delta_F = 1$ if F_i acts in the direction of the process and $\delta_F = -1$, if it opposes the evolution of the process.

The critical state is reached when,

$$P_T = P_{cr}(t), \qquad (12)$$

where the critical participation, a dimensionless variable, is given by the eq.,

$$P_{cr}(t) = P_{cr}(0) - D_T(t) - P_{res}. \qquad (13)$$

For crackless structures and no residual stresses $P_{cr}(t) = P_{cr}(0)$ such as, in this case, the group of static loads becomes critical if,

$$\sum_i (F/F_{cr})_i^{\alpha+1} \cdot \delta_F = P_{cr}(0). \qquad (14)$$

If a tubular specimen is under loads p, F and M_b (fig. 2) eq. (14) becomes,

$$\left(\frac{p}{p_{cr}}\right)^{\alpha+1} + \left(\frac{F}{F_{cr}}\right)^{\alpha+1} \cdot \delta_F + \left(\frac{M_b}{M_{b,cr}}\right)^{\alpha+1} \cdot \delta_M = P_{cr}(0), \qquad (15)$$

where $\delta_M = 1$ in the section where M_b causes elongation and $\delta_M = -1$ in the section where M_b produces compression, $\delta_F = 1$ if F produces elongation and $\delta_F = -1$ if F produces compression.

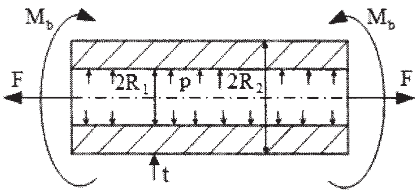

Fig. 2.Tubular specimen loaded with internal pressure, p, axial force, F, and bending moment M_b

In interpreting the experimental data, in general, one can see that $P_{cr}(0) \neq 1$ is a random value. Consequently, the critical group is not a single value but a stochastic distribution between $P_{cr,\,min}(0)$ and $P_{cr,\,max}(0)$, depending on the probability of structure material failure. These justify the scatter of experimental data. Several test points may be outside the upper ($P_{cr,max}(0)$) and lower ($P_{cr,min}(0)$) bounds. This may be caused by inaccuracy of material property and experimental measurement. Therefore, one can conclude that the proposed criterion (15) for critical group of loads is an effective criterion for the fracture of defect – free tubular sample. It can be used in the engineering design and integrity assessment of tubular sample.

In the case of simultaneous loading with internal pressure and bending moment eq. (15) becomes,

$$\left(\frac{p}{p_{cr}}\right)^{\alpha+1} + \left(\frac{M_b}{M_{b,cr}}\right)^{\alpha+1} \cdot \delta_M = P_{cr}(0). \qquad (16)$$

For linear-elastic loading ($\sigma_{max} \leq \sigma_y$) $\alpha = 1/k = 1$. If one adds to the above $P_{cr}(0)=1$, corresponding to the use of deterministic values of the mechanical characteristics, then relationship (16) becomes,

$$\left(\frac{p}{p_{cr}}\right)^2 + \left(\frac{M_b}{M_{b,cr}}\right)^2 = 1. \qquad (17)$$

This relationship was obtained experimentally [13] with tube specimens made from carbon steel (St 20) and austenitic steel (12X18H10T).

Equation (17) has been proposed for calculating the resistance of pipes featuring $\beta = R_2/R_1 \leq 3$, simultaneously loaded with internal pressure p and bending moment M_b [13]. For critical parameters the following relations have been proposed [14]

$$\left.\begin{aligned} p_{cr} &= \frac{2}{\sqrt{3}} \cdot \sigma_{0.2} \cdot \ln\beta; \\ M_{b,cr} &= 2 \cdot \sigma_{0.2} \cdot S_s, \end{aligned}\right\} \qquad (18)$$

which is the yield pressure and the yield bending moment, respectively. Stress $\sigma_{0.2} = \sigma_y$ is the yield stress corresponding to a residual strain of 0.2% while S_s is the static moment of the section area.

Strength of tubular branch junction

In many practical cases the tubular joints welds are susceptible to crack formation because the nozzle - vessels junction often experience mixed mode loading (fig. 3). Consequently the strength is affected by the cracks.

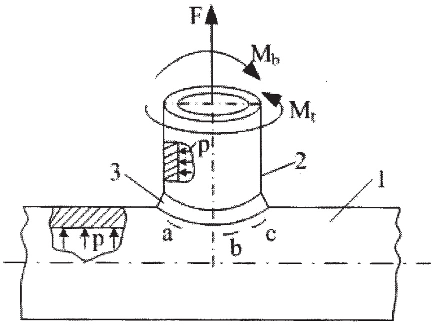

Fig. 3. Tubular junction: 1 – run pipe; 2 – branch pipe; 3 – weld; a, b, c – cracks

The state of a cracked tubular joint subjected to internal pressure, p, bending moment, M_b, torsion moment, M_t, tension force, F, becomes critical if eq. (12) is fulfilled, namely,

$$\left(\frac{p}{p_{cr}}\right)^{\alpha+1}+\left(\frac{F}{F_{cr}}\right)^{\alpha+1}\cdot\delta_F+$$
$$+\left(\frac{M_b}{M_{b,cr}}\right)^{\alpha+1}\cdot\delta_M+\left(\frac{M_t}{M_{t,cr}}\right)^{\alpha_1+1}=P_{cr}(t). \quad (19)$$

In the case of internal pressure and bending moment loading, neglecting the residual stress influence, and limiting the maximum stress to the value of yield stress (when k = 1 and $\alpha = 1 / k = 1$), eq. (19) becomes,

$$\left(\frac{p}{p_{cr}}\right)^2+\left(\frac{M_b}{M_{b,cr}}\right)^2=P_{cr}(t), \quad (20)$$

where $P_{cr}(t) = P_{cr}(0) - D_T(t)$.

Ignoring the deterioration will lead to overestimation the resistance of the branch junction.

In the cases when the critical loads are deterministic values ($P_{cr}(0)=1$),

$$P_{cr}(t)=1-D_T(t) \quad (21)$$

One uses eqs. (20) and (21) to evaluate the results obtain in the papers [15÷18].

In the paper [15] a branch junction were used (fig. 4), which had a branch/run pipe mean diameter ratio r_m / R_m = 0.5, a thickness ratio t / T = 1.0 and diameter/thickness ratio $2R_m$ / T = 20.

Through-wall cracks were considered (T = t) with an angular extension 2β [15; 16]. The critical loads were assumed the limit loads, corresponding to yield stress.

For uncracked model ($D_T(t) = 0$) the interaction diagram is a circle with the radius $\sqrt{P_{cr}(t)}=1$ (fig. 5).

If 2β > 0 then $D_T(t) > 0$ and eq. (20) describes circles with the radius $\sqrt{P_{cr}(t)}<1$ (fig. 5). For 2β = 49° results a circle with the radius $\sqrt{P_{cr}(t)}=0.957$. For 2β = 95° results a circle with the radius $\sqrt{P_{cr}(t)}=0.883$, while for 2β = 140° results a circle with the radius $\sqrt{P_{cr}(t)}=0.8$.

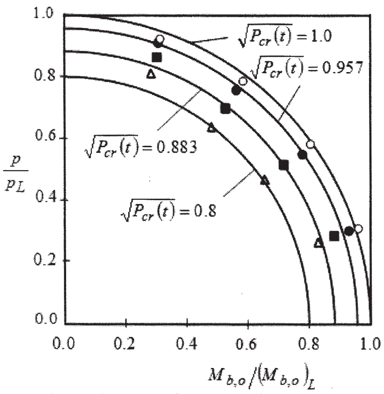

Fig. 5. Interaction diagram for combined pressure and bending moment loading of branch junction for 2Rm = 20; r_m / Rm = 0.5; t = T (through wall crack): o uncracked;• 2β = 49°; ■ 2β = 95°; Δ 2β = 140°; [15; 16]

Similar results were obtained for $2R_m$ / T = 10 and 30 and with r_m / R_m = 0.95 [17].

The paper [18] describes the effect of cracks on the limit loads of a branch junction under combined pressure and bending to the branch pipe and, separate, to the run pipe. Two branch components were considered, namely (fig. 4):

- large bore with R_m = 244.5 mm; T = 31.8 mm; r_m = 154 mm; t = 15.9 mm.
- medium bore with R_m = 222.5 mm; T = 20 mm; r_m = 59.5 mm; t = 8 mm.

For un-cracked branch junction under combined pressure and bending to the branch piping, the interaction curve (p / p_{cr} versus $Mb / M_{b,cr}$) is circular, given by eq. (20) with $D_T(t) = 0$,

$$\left(\frac{p}{p_{cr}}\right)^2+\left(\frac{M_b}{M_{b,cr}}\right)^2=1. \quad (22)$$

For through-wall crack, a = T, with the crack located on the crotch in the lower weld toe (fig. 4), with relative length $\alpha / \pi \le 0.5$ the circular interaction results as in figure 6; it is described by eqs. (20) and (21) where $D_T(t) = D(a, \alpha)$ is the deterioration due to crack. The circles are of radius $\sqrt{P_{cr}(t)}=\sqrt{1-D(a;\alpha)}$, in both cases: large bore (fig. 6, a) and medium bore (fig. 6, b). One can see that the points obtained in [18] fits with eq. (20) for $\sqrt{P_{cr}(t)}$ =0.8 and 0.9 in the case of large bore branch pipe and for $\sqrt{P_{cr}(t)}=0.84$ and 0.92 in the case of medium bore branch pipe. Because $\sqrt{P_{cr}(t)}$ for medium bore, for the same α / π, is higher than for large bore, one may conclude: the medium bore junction has a higher strength then the large bore junction.

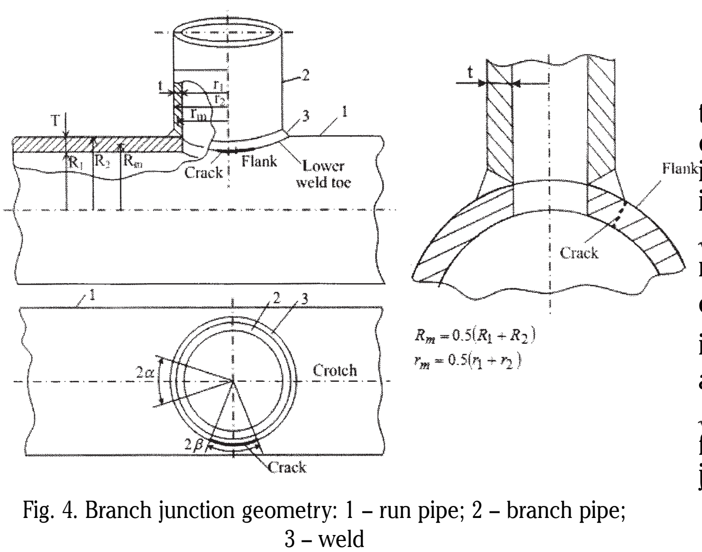

Fig. 4. Branch junction geometry: 1 – run pipe; 2 – branch pipe; 3 – weld

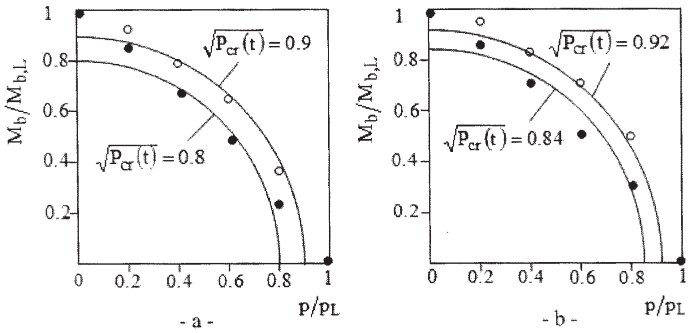

Fig. 6. Through-wall cracked branches (a = T) located in the lower weld toe (fig. 4) under pressure and in-plane bending moment to the branch pipe: a – large bore branch; b – medium bore branch. The curves with eq. (20) and the points after [18]: $\alpha / \pi = 0.25$ (o) and $\alpha / \pi = 0.25$ (■).

Conclusions

One analysis the critical stresses and critical loadings for specimens without and with cracks. On this basis strength calculation methods are proposed:

- the classical method based on the equivalent stress;
- the method based on the principle of critical energy.

The concept of specific energy introduced by principle of critical energy is used for strength evaluation of: - tubular sample loaded by internal pressure, axial force and bending moment; - tubular branch junction loaded by internal pressure, bending moment, axial force and torsional moment.

The analytical eqs. obtained on the basis of principle of critical energy were verified against data from literature for cracked branch junctions. The theoretical results are in good agreement with the data reported in literature.

References

1. JINESCU V.V., Principiul energiei critice şi aplicaţiile sale (The Principle of Critical Energy and its Applications), Editura Academiei Române, Bucureşti, 2005.
2. JINESCU V.V., Critical Energy Approach for the Fatigue life Calculation under Blocks with different normal Stress Amplitudes, Int. J. Mech. Sci., **67**, 2013, p. 78.
3. STRIJALOV V.A., Ţicliceskaia procinosti i polzucesti metalov pri malotiklovom nagrujenii v usloviiah nizkih i vîsokih temperature, Naukov Dumka, Kiev, 1978.
4. JINESCU V.V., IORDĂCHESCU V.I., Calculation of deterioration due to cracks in tubular specimens, U.P.B. Sci. Bull., Series D, **76**, Iss. 1, 2014, p. 149.
5. JINESCU V.V., IORDĂCHESCU V. I., TEODORESCU N., Relation for Calculation of Critical Stresses in Pressure Equipment with Cracks, Rev. Chim. (Bucharest), **64**, no. 8, 2013, p. 858.
6. JINESCU V.V., Traitise on Thermomechanics, vol. 1, Ed. AGIR, Bucharest, 2011.
7. KIM Y-J, SHIM D-J, NIKBIN K., KIM Y-J, HWANG S-S, KIM J-S, Finite element based plastic limit loads for cylinders with part-through surface cracks under combined loading, Int. J. of Press. Vess.&Piping, **80**, no. 7-8, 2003, p. 527.
8. KIM N-H, OH C-S, KIM Y-J, KIM J-S, JERNG DW, BUDDEN PJ, Limit loads and fracture mechanics parameters for thick-walled pipes, Int. J. of Press. Vess.&Piping, **88**, no. 10, 2011, p. 403.
9. OH C-S, SONG T-K, KIM Y-J, Approximate *J* estimates for axial part-through surface cracked pipes, Fat. Fract. Eng. Mater. Struct., **30**, no. 12, 2007, p. 1127.
10. JILIUKAS A. IU., Razrusenie konstrukţionnîh elementov, Mokslas, Vilinius, 1988.
11. SIRATORI M., MIESI T., MAŢUSITA H., Vîcislitelinaia mehanika razrusenia, pod redakţii E.M. Morozova, Moskva, 1986.
12. ANDREIKIV AE, IVANIŢKII IA.L, ZAZULIAK VA, VARIVODA IU., Prognozirovanie hrupkoi procinosti elementov konstruckţii v usloviiah slojnovo napriajennovo sostoianiia, Tez. Dokl., Vsesoiuznovo simpoziuma po mehanike razruşeniia, Kiev, 1985.
13. SERGHIEV B.P, GUSEV B.M, Issledovanie nesuşcei sposobnosti tolstostennîh truboprovodov pri sovmestnom deistvii davleniia i izghibaiuşcevo momenta, in Himiceskoe i neftianoe maşinostroenie, **9**, 1979, p. 15.
14. SERGHIEV B.P, Issledovanii predelinoe sostoianiia tolstostenîh turboprovodov pri odnovremennom deistvii izghibanişcevo momenta i davleniia, in Himiceskoe i neftianoe maşinostroenie, **12**, 1976, p. 12.
15. MOFFAT D.G., Limit loads for cracked piping components, Reference Stress Methods. Analysing Safety and Design, Editor Ian W. Goodall, Professional Engineering Publishing, London, 2003.
16. LYNCH M. A., MOFFAT D. G., MORETON D. N., Limit loads for a cracked branch junction under pressure and branch out-of-plane bending, Int. J. Press. Vess.&Piping, **77**, no. 4, 2000, p. 185.
17. LYNCH M. A., Limit loads of piping branch junctions with cracks, Phd Thesis, The University of Liverpool, 2001.
18. MYEONG M. S., KIM Y. J., BUDDEN P. J., Limit load interaction of cracked branch junctions under combined pressure and bending, Eng. Fract. Mech., **86**, 2012, p. 1

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